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An improved forecasting approach to reduce inventory levels in decentralized supply chains

Youssef Tliche*, Atour Taghipour, Béatrice Canel-Depitre

[a] University of Le Havre Normandie, Postal address: 25 Rue Philippe Lebon, 76600 Le Havre, France

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ABSTRACT

This paper covers forecast management in decentralized supply chains. For various reasons, companies do not always agree to disclose their information. To deal with this issue, we consider a downstream demand inference (DDI) strategy in a two-level supply chain. DDI was assessed using different forecasting methods and was successfully tested using only a simple moving average. In an investigatory context using other forecasting methods, we propose the introduction of the weighted moving average method, which affects nonequal weights to past observations. First, we verify the unique propagation of demand processes. Second, we consider the forecast mean squared errors, the average inventory levels and the bullwhip effect as the supply performance metrics. Third, we formalize the manufacturer's forecast optimization problem and apply Newton's method to solve it. The optimization results, based on the simulated demands, confirm the effectiveness of our approach to produce further enhanced solutions and to improve the results of DDI. We have shown that a little change in the weights of the forecast method improves the competitiveness in the market. Conversely, the bullwhip effect is affected due to the nonequal weighting in the forecast method.

1. Introduction

The optimal supply chain performance requires the realization of numerous actions. Regrettably, those actions are not always in the best interest of the actors of the same supply chain. The supply chain actors are mainly focused on achieving their own objectives and that self-serving focus often leads to poor performance. However, enhanced performance is achievable if the companies coordinate their operations such that each company's objectives become aligned with the supply chain's performance.

Supply chain management is one of the most important research areas that aims to improve the overall supply chain performance. More specifically, actors in supply chains are continuously seeking to minimize their inventory levels, which are translated into cost savings over time. In a decentralized two-level supply chain consisting of a manufacturer and a retailer, the manufacturer seeks to improve the quality of the forecast MSE with the objective of minimizing his average inventory levels. In fact, information sharing presents a common approach to deal with inventory reductions. On one side, a stream of papers (Cachon & Fisher, 2000; Sahin & Robinson, 2005; Yu, Yan & Edwin Cheng, 2001) argued that information sharing can reduce the inventory holdings, related costs and bullwhip effect occurring in supply chains. Conversely, many researchers (Fawcett, Osterhaus, Magnan, Brau & McCarter, 2007; Forslund & Jonsson, 2007; Klein, Rai & Straub, 2007; Lee & Whang, 2000; Mendelson, 2000) also argued that information sharing has a number of practical limitations, such as confidential policies, data reliability and the lack of information systems' compatibility. Vosooghidizaji, Taghipour and Canel-Depitre (2019) considers different scenarios wherein asymmetric information cannot be shared with supply chain partners because of many reasons that include "the fear of losing competitive advantage, getting extra benefits, getting a better price, maintaining one's bargaining power, not being controlled or dictated to by other parties, ensuring compatibility of information systems, and other strategic reasons". An actor not sharing information can affect the whole system of the supply chain. It was also argued that, by revealing sensitive demand information to the upstream manufacturer, a retailer may lose some advantage in future price negotiations (Ha, Tong & Zhang, 2010). Wal-Mart announced that it would no longer share its information with other companies like Inc and AC Nielson as Wal-Mart considers data to be a top priority and fears information leakage (Hays, 2004). In fact, depending on the nature and size of supply chains, not sharing information can result to differ-

* Corresponding author.
E-mail addresses: youssef.tliche@univ-lehavre.fr, tliche.youssef@yahoo.com (Y. Tliche), atour.taghipour@univ-lehavre.fr (A. Taghipour), beatrice.canel@univ-lehavre.fr (B. Canel-Depitre).

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ent levels of losses. William Wappler, President of Automotive Technology Leader SURGER, says, "The automotive industry is estimated to lose annually more than 2 billion dollars in the supply chain due to losses in inventory of containers, parts, finished vehicles and logistical inefficiencies, through a notorious lack of visibility and inherent control." He adds: “Most automotive companies struggle to reduce supply chain costs year over year.” According to their internal forecasts, the use of the proposed digital platform can help participants achieve double-digit cost savings through highly accurate supply chain visibility and the collaborative power of shared information (Henderson, 2018). Indeed, industries where component suppliers need to build a high capacity in advance due to short lead times, face high inventory costs because of uncertain market demand. Generally, it has been accepted that the demand is a private information of the retailers, that leads to problems of management of the inventory at the upstream levels.

Recently, a new coordination supply chain approach, known as downstream demand inference (DDI), (Ali & Boylan, 2012; Ali & Boylan, 2011; Ali, Babai, Boylan & Syntetos, 2017; Tliche, Taghipour & Canel-Depitre, 2019) emerged in the supply chain field. The DDI strategy allows the enhancement of decentralized systems without having to go through explicit demand information sharing. Instead of demand information sharing, the upstream actor can infer the demand from the order history. The DDI strategy assumes that the demand process and its parameters are known throughout the two-level supply chain. The first part of the assumptions – that is the retailer facing the customer’s demand is able to easily estimate the parameters of the process from his demand history – is evident. The second part of the assumptions – that is the ability of the manufacturer to infer the process of the demand occurring at the retailer – is subject of research and discussion in the literature. Ali and Boylan (2011) showed that DDI cannot be applied with the optimal minimum mean squared error (MMSE) forecast method because the propagation of the demand may not be unique. Ali and Boylan (2012) also showed that DDI is not possible with the single exponential smoothing (SES) method, but only when the downstream actor uses the simple moving average (SMA) method that attaches equal weights to past observations. Ali et al. (2017) showed that DDI generally outperforms the no information sharing (NIS) strategy in terms of the forecast’s mean squared error (MSE) and inventory costs under the assumption of an $AR(1)$ demand model. Under the DDI strategy, Tliche et al. (2019) considered the MSE^{DDI} and average inventory level ($\tilde{I}_t^{DDI}$) as upstream supply chain performance metrics and generalized the above results for causal invertible[1] $ARMA(p, q)$ demand processes. In a context of DDI strategy, this paper aims to further enhance the DDI’s results by acquiring further optimized solutions in terms of MSE and average inventory levels.

A first possibility for improvement can be emphasized on the forecasting method adopted in the DDI approach. Since the SMA method is the only method of prediction up to now allowing the inference of the downstream demand, we have thought to introduce a variant of this method while keeping in mind the orientation of improvement of the average inventory levels. The SMA method is characterized by the equal weights associated to the N past observations, to predict the future demand. Every time period, the oldest demand observation is dropped out and exchanged by the last demand observation. A first intuition is to disrupt the weightings of the method in order to improve the performance. In this way, the Weighted Moving Average (WMA) method was selected in order to first investigate its feasibility in the DDI approach, and second to investigate whether any enhancement is achievable in a two-level supply chain. The WMA method is a simple forecasting method, as well as the SMA method. The WMA method attaches different weights/ponderations to the N past demand observations, and in the same way as SMA, the oldest demand observation is dropped out and exchanged by the last demand observation, every time period. The disruption’s possibility of the weights in the method was an opening door for exploring potential improvements in different directions. One of these directions is the optimization according to the upstream actor’s average inventory levels.

Consequently, acquiring further “optimized” solutions naturally opens the line of our research to other branches of scientific research. Indeed, optimization plays a very important role in several areas of application and especially in supply chain management. Omnipresent since the beginning of time, optimization is a mathematical discipline, that has grown in importance during the 20th century. This is due to the development of industrial sciences, operations planning (economics, management, logistics, scheduling), emerging technologies (automatic, electronic, electrotechnical, etc.) and computer science, which has made previously impassable numerical resolution methods efficient. Mathematically, it consists of minimizing, or maximizing, a function that represents an objective to be achieved on a set called a "domain" or "set of feasible solutions," which is defined as a set of constraints that are to be respected. The objective is to find the best solution belonging to the domain that acquires the optimal value of the objective function. The nature of the objective function and the constraints defining the domain determine the nature of the optimization problem and the difficulty of its resolution.

In this paper, instead of using the SMA method, we use the weighted moving average (WMA) method, which attaches different weights to the N past observations, and then re-establish the manufacturer MSE^{DDI} and $\tilde{I}^{DDI}$ expressions according to a weighting vector x. Second, we propose two measures to quantify the gap separating the adoption of the NIS strategy with the MMSE method to the adoption of the DDI strategy with the WMA method on one hand, and on the second hand to quantify the gap separating the adoption of the DDI strategy with the SMA method to the adoption of the DDI strategy with the WMA method, in terms of bullwhip effect. Third, we mathematically formalize the manufacturer’s forecast optimization problem (MFOP) and propose the application of the well-known Newton’s method in order to obtain the optimal weighting vector x^*. To the best of our knowledge, this paper presents the first attempt to introduce Newton’s method into forecasts where the WMA method is adopted. The numerical results of the MSE and $\tilde{I}_t$ optimizations based on the simulated causal invertible $ARMA(p, q)$ demand processes confirm the effectiveness of this approach to produce further-optimized solutions compared to NIS strategy with MMSE method and DDI strategy with SMA method, and consequently to improve the competitiveness in the market. However, the WMA method affects the bullwhip effect since nonequal weights generate higher orders’ variability. It is concluded that if the supply chain is initially adopting a NIS strategy where the MMSE method is used in the downstream forecasts, the downstream actor is emphasized to consider a high value of N (beyond a certain break-point) in order to reduce the bullwhip effect. Else, if the supply chain is initially adopting a DDI strategy where the SMA method is used in the downstream forecasts, the upstream actor needs to use a reserve inventory in order to cover the amplified orders variations. Hence, we provide a developed picture of the DDI strategy’s adoption when the WMA is used for demand forecasts and where the Newton’s method is employed to quantify the weighting vector of the WMA method, according to the minimization of the upstream average inventory levels.

The rest of the paper is organized as follows. Section 2 is devoted to the literature review. In Section 3, we present

[1] Please refer to Shumway and Stoffer (2011) for more details on $ARMA$ models, causality and invertibility.

the proposed modeling approach. Section 4 is devoted to the implementation, simulation and discussions. Finally, in Section 5, we summarize the contributions, the results, the limitations and perspectives.

2. Literature review

Demand information sharing is one of the most important catalysts leading to improvements in supply chains. Sharing customers' demand information requires that upstream actors have access to the demand data of their respective downstream actors. The need for information sharing mechanisms, in order to extract demand information has always been an open topic of discussion in the literature. Some researchers (Chen, Drezner, Ryan & Simchi-Levi, 2000b; Lee, So & Tang, 2000; Raghunathan, 2003; Yu, Yan & Cheng, 2002) argued that downstream actors need to share their demand information with upstream actors in order to reduce the bullwhip effect. On the other hand, other researchers (Gaur, Giloni & Seshadri, 2005; Gilbert, 2005; Raghunathan, 2001; Zhang, 2004) relied on some strong arguments to show that the received orders already contain information about the customers' demand process.

In a context of no information sharing policy, DDI appears to be a novel collaboration management approach that allows the upstream actor to infer the demand of his formal downstream actors without the need for information sharing mechanisms. According to the DDI approach, the inventory level/cost savings from coordination and negotiation are possible if trust is established between parties (Ali et al., 2017; Tliche et al., 2019).

The works of Ali and Boylan (2011) and Ali and Boylan (2012) have already shown that "DDI is not possible through SES or optimal MMSE methods, but only with nonoptimal SMA method". This is due to the nonfeasibility of DDI when the propagation of the demand throughout the supply chain is not unique. Ali et al. (2017) have investigated DDI using the SMA method for an $AR(1)$ demand model and conducted numerical analysis based on real data. Based on simulations, Tliche et al. (2019) generalized DDI's results for causal invertible $ARMA(p, q)$ demand models and showed that this strategy reduces the bullwhip effect. Consequently, it is still natural to explore the feasibility of DDI and the improvement of the results by using other forecasting methods. It's first about the margin of enhancement still existing between the DDI strategy's results and the forecast information sharing (FIS) strategy's results which corresponds to the centralized system where the demand information is explicitly shared between actors. Therefore, exploring the DDI strategy by adopting simple forecasting methods is still an interesting management research area for both researchers and practitioners.

Forecasting in supply chains is an increasingly critical organizational tool (Sanders & Manrodt, 2003) for improving business competitiveness. Ali and Boylan (2012) provided a summary of the highly ranked forecasting methods according to their usage, familiarity and satisfaction among practitioners. Generally, supply chain decision-makers choose a forecasting method based on its simplicity. Especially, the SES, regression analysis (RA) and SMA methods are popular among forecasting managers for familiarity and satisfaction reasons. As reported in the works of Sanders and Manrodt (1994) and Boylan and Johnston (2003), because of their high difficulty and sophistication, optimal forecasting methods are most often considered to be undeserving of extra effort. On the other hand, nonoptimal forecasting methods are more intuitive, especially for those with limited mathematical backgrounds. In addition, Johnston, Boyland, Meadows and Shale (1999) showed that "the variance of the forecast error for the nonoptimal method SMA was typically 3% higher than the SES method for an $ARIMA(0, 1, 1)$".

In this paper, we examine the effects of employing a simple nonoptimal forecasting method, namely, the WMA, in the downstream actor's forecasts, where demand follows a causal invertible $ARMA(p, q)$. The WMA is a method that is widely used in the industry literature (Alsultanny, 2012; Eckhaus, 2010; Kalaoglu et al., 2015; Kapgate, 2014; Wang & Cheng, 2007; Wenxia, Feijia, Shuo, Kun & Guodong, 2015). We selected the WMA method as a method of interest because it belongs to the moving average methods, and more specifically a variant of the SMA method. The narrow difference in weights between the SMA and the WMA methods suggested a potential feasibility (uniqueness of demand process propagation) of the DDI approach in a decentralized supply chain. Such as SMA method, the WMA method is based on the shifting forward of the last N observations in order to predict the future. Every time-period, the oldest observation is excluded and the most recent observation is included.

Table. 1 summarizes the experimented forecasting methods in the context a DDI strategy as well as our contribution.

Table 1
Forecasting when DDI strategy is adopted.

Forecasting method	Mathematical expression	DDI feasibility	Reference
MMSE	$\boldsymbol{f_{t+1} = E(D_{t+1}/\{D_t, D_{t-1}, \ldots, D_{t-T}\})}$ • $\{D_t, D_{t-1}, \ldots, D_{t-T}\}$ is the available set of demand history	Not feasible	Ali and Boylan (2011)
SES	$\boldsymbol{f_{t+1} = \alpha \sum_{j=0}^{T} (1-\alpha)^j D_{t-j}}$ • $\{D_t, D_{t-1}, \ldots, D_{t-T}\}$ is the available set of demand history • α is the smoothing constant	Not feasible	Ali and Boylan (2012)
SMA	$\boldsymbol{f_{t+1} = \frac{1}{N} \sum_{j=0}^{N-1} D_{t-j}}$ • N is the moving average horizon	Feasible	Ali and Boylan (2012) Ali et al. (2017)
WMA	$\boldsymbol{f_{t+1} = \sum_{i=1}^{N} x_i \, D_{t+1-i}}$ • N is the moving average horizon • $x = (x_1, \ldots, x_N)$ is the weighted vector that is obtained using Newton's method • $\begin{cases} \sum_{i=1}^{N} x_i = 1 \\ x_i \geq 0 \ \forall i = 1, \ldots, N \end{cases}$	Feasible	This paper

Thus, in this paper, we first show that demand inference is feasible when the retailer uses the WMA method in his forecasts. The upstream actor is then able to infer the demand arriving at his formal downstream actor as the demand propagation is unique. Next, the consideration of nonequal weights for the N past observations in the WMA method, is achieved through the use of the Newton optimization method aligned according to the minimization of the MSE and thus according to the minimization of the upstream actor's average inventory levels. In this way, this paper provides a methodology that allows the reduction of the average inventory level at the upstream actor. Since there are no specific "standard approaches" for determining the optimal setting in terms of parameter N and lead-time L, we study the sensitivity of our approach's results in comparison with the NIS strategy through the MMSE method, and in comparison with the DDI strategy through the SMA method. In addition, we present findings on the bullwhip effect in order to obtain a clearer picture of this approach.

3. Modeling approach

We consider a simple two-level supply chain that is formed by a manufacturer (upstream actor) and a retailer (downstream actor) who receives the demand of a final customer. We suppose that a periodic review system is adopted for replenishment in which downstream actors place their orders with upstream actors after examining their respective inventory levels. Indeed, after the realization of demand D_t by the retailer at the beginning of time period t and after checking his own inventory level, the retailer places an order Y_t before the end of the period. Then, the manufacturer prepares the required order Y_t and ships it to the retailer who will receive it at period $t+L+1$. Here, L presents the replenishment time of both production and shipment. Second, it is assumed that there are no order costs. Second, the unit inventory holding costs and shortage costs are constant and respectively denoted by h and s. It is also assumed that both the manufacturer and retailer adopt an order-up-to (OUT) policy, which minimizes the total costs over an infinite time horizon (Lee et al., 2000).

These assumptions were adopted in many papers of this stream of research (Ali et al., 2017; Ali, Boylan & Syntetos, 2012; Hosoda et al., 2008; Hosoda and Disney, 2006; Cheng and Wu., 2005; Alwan et al., 2003; Chen et al., 2000; Lee et al., 2000; Raghunathan, 2001; Tliche et al., 2019) and we consider our paper is part of the continuity of this stream of works.

3.1. Customer's demand model and forecast method

Time-series processes have widely been adopted to model the demand of many products in different fields. Let us assume that the demand at the retailer is a causal invertible $ARMA(p,q)$ process. Let D_t be this demand process at period t, which is expressed by equation (1) as follows:

$$D_t = c + \sum_{j=1}^{p} \phi_j D_{t-j} + \xi_t + \sum_{j=1}^{q} \theta_j \xi_{t-j} \tag{1}$$

where

$c \geq 0$ is the unconditional mean of the demand process,

ϕ_j where $j \in \{1, .., p\}$ is the autoregressive coefficient of the demand process,

θ_j where $j \in \{1, .., q\}$ is the moving average coefficient of the demand process, and

$\xi_t N \to (0, \sigma_\xi^2)$ where $t \in [0, +\infty]$ is the independent and identically distributed error term that follows a normal distribution.

Furthermore, let d_t be the mean-centered demand process, μ_d be the unconditional mean of the demand process D_t and $\gamma_k = Cov(D_{t+k}, D_t)$ be the covariance between demands at periods t and $t+k$. These definitions are required for the formulas' derivations in this work.

In addition, as mentioned above, we will consider that the retailer adopts the WMA method in the demand forecasts, which, at period $t+1$, is mathematically written as equation (2):

$$f_{t+1} = \sum_{i=1}^{N} x_i \, D_{t+1-i} \tag{2}$$

where x_i is the weight that is associated with the customer's demand occurring at time period $t+1-i$, which verifies the set of constraints $(C): \begin{cases} \sum_{i=1}^{N} x_i = 1 \\ x_i \geq 0 \ \forall i \in \{1, \ldots, N\} \end{cases}$, and let $x = \begin{pmatrix} x_1 \\ \vdots \\ x_N \end{pmatrix}$ be the weighting vector.

To apply DDI strategy, it is first important to check whether the propagation of the demand across the supply chain is unique.

3.2. Downstream actor's orders time-series structure

Let Y_t be the order process arriving at the manufacturer at period t, which is expressed by equation (3) as follows:

$$Y_t = c + \sum_{j=1}^{p} \phi_j Y_{t-j} + \tilde{\xi}_t + \sum_{j=1}^{q} \theta_j \tilde{\xi}_{t-j} \tag{3}$$

where

$c \geq 0$ is the unconditional mean of the order process,

ϕ_j where $j \in \{1, .., p\}$ is the autoregressive coefficient of the order process,

θ_j where $j \in \{1, .., q\}$ is the moving average coefficient of the order process, and

$\tilde{\xi}_t \to N(0, [L^2(x_1^2 + x_N^2 + \sum_{i=1}^{N-1} (x_{i+1} - x_i)^2) + 2Lx_1 + 1] \, \sigma_\xi^2)$ where $t \in [0, +\infty]$ is the independently and identically distributed error term that follows a normal distribution.

The demand and order processes have the same autoregressive and moving average coefficients, and they differ only by their respective error terms (see Appendix A). Indeed, the order's error terms are amplified by a coefficient $\beta = L^2(x_1^2 + x_N^2 + \sum_{i=1}^{N-1} (x_{i+1} - x_i)^2) + 2Lx_1 + 1$ such as $\sigma_{\tilde{\xi}}^2 = \beta \sigma_\xi^2$. Consequently, the order process is unique and the upstream actor is able to infer the demand process without the need for demand information sharing. Next, we derive the manufacturer's forecast MSE^{DDI} and $\tilde{I}^{DDI}$ when the WMA method is used in a context of a DDI strategy. The performance metrics MSE^{DDI} and $\tilde{I}^{DDI}$ are considered since they are the first direct measures impacted by demand inference. Indeed, the upstream actor benefits from the DDI strategy that enables the reduction of the MSE and the average inventory level. The next consequence is then the reduction of the inventory costs related to these metrics' enhancements.

3.3. Derivation of the manufacturer's mean squared error and average inventory level expressions

Since the forecast expression in equation (2) is a function of the weights, the MSE^{DDI} and $\tilde{I}_t^{DDI}$ expressions are also functions of these weights. We derive the $MSE^{DDI}(x)$ and $\tilde{I}_t^{DDI}(x)$ expressions as follows:

$$MSE^{DDI} = Var\left[\sum_{i=1}^{L+1} (D_{t+i} - f_{t+i})\right] = Var\left[\sum_{i=1}^{L+1} D_{t+i} - (L+1) f_{t+1}\right]$$

$$\Leftrightarrow \ MSE^{DDI} = Var\left(\sum_{i=1}^{L+1} D_{t+i}\right) + (L+1)^2 Var(f_{t+1})$$

$$-2(L+1)Cov\left(\sum_{i=1}^{L+1} D_{t+i}, f_{t+1}\right) \tag{4a}$$

We then derive the three components of Eq. (4a) (see Appendix B) and obtain the final expression of Eq. (4) as follows:

$$\begin{aligned} \boldsymbol{MSE^{DDI}}(\boldsymbol{x}) &= (\boldsymbol{L}+1)\boldsymbol{\gamma}_0 + 2\sum_{\boldsymbol{i}=1}^{\boldsymbol{L}} \boldsymbol{i}\ \boldsymbol{\gamma}_{\boldsymbol{L}+1-\boldsymbol{i}} \\ &+ (\boldsymbol{L}+1)^2\left[\boldsymbol{\gamma}_0\sum_{\boldsymbol{i}=1}^{\boldsymbol{N}} \boldsymbol{x_i}^2 + 2\sum_{\boldsymbol{j}=1}^{\boldsymbol{N}-1}\left(\boldsymbol{x_j}\sum_{\boldsymbol{i}=\boldsymbol{j}+1}^{\boldsymbol{N}} \boldsymbol{x_i}\boldsymbol{\gamma}_{\boldsymbol{i}-\boldsymbol{j}}\right)\right] \\ &- 2(\boldsymbol{L}+1)\sum_{\boldsymbol{i}=1}^{\boldsymbol{L}+1}\sum_{\boldsymbol{j}=1}^{\boldsymbol{N}} \boldsymbol{x_j}\boldsymbol{\gamma}_{\boldsymbol{i}+\boldsymbol{j}-1} \end{aligned} \tag{4}$$

Next, the general expression of the average inventory level under an OUT policy is given by Ali et al. (2012) and mathematically written as Eq. (5a) as follows:

$$\tilde{I}_t = T_t - E\left(\sum_{i=1}^{L+1} Y_{t+i}\right) + \frac{E(Y_t)}{2} \tag{5a}$$

where Y_t is the order process of the retailer arriving at the manufacturer at time period t; the manufacturer's optimal OUT inventory level T_t is expressed by $T_t = M_t + K\sigma_{\breve{\xi}}\sqrt{V}$, where M_t and V are respectively the conditional expectation and the conditional variance of the total demand over the lead-time plus one review time unit; and $K = F^{-1}_{N(0,1)}(\frac{s}{s+h})$ is the inverse distribution function for the standard normal distribution that is calculated at the ratio point $\frac{s}{s+h}$.

Consequently, under the DDI strategy and using the WMA method for the demand forecasts, we obtain Eq. (5b) as follows:

$$\tilde{I}_t^{DDI}(x) = T_t^{DDI}(x) - E\left(\sum_{i=1}^{L+1} Y_{t+i}\right) + \frac{E(Y_t)}{2} \tag{5b}$$

where $T_t^{DDI}(x) = M_t{}^{DDI}(x) + K\sigma_{\breve{\xi}}\sqrt{V^{DDI}(x)}$

Then, the Eq. (5b) is equivalent to the following Eq. (5c):

$$\tilde{I}_t^{DDI}(x) = M_t{}^{DDI}(x) + K\sigma_{\breve{\xi}}\sqrt{V^{DDI}(x)} - E\left(\sum_{i=1}^{L+1} Y_{t+i}\right) + \frac{E(Y_t)}{2} \tag{5c}$$

We then derive the four components of equation (5*c*) (see Appendix C), and thus, we obtain the final expression of equation (5) as follows:

$$\tilde{I}_t^{DDI}(x) = \frac{c}{2\left(1-\sum_{j=1}^{p}\phi_j\right)} + K\sigma_{\breve{\xi}}\sqrt{MSE^{DDI}(x)} \tag{5}$$

We note that $\tilde{I}_t^{DDI}(x)$ in equation (5) is a nonlinear function of $MSE^{DDI}(x)$, which can explain the nonproportional evolution linking these two performance metrics. Next, we proceed to deriving the resulting bullwhip effect in order to compare the processes variations' evolution with the cases where the NIS strategy with the MMSE is adopted, and then to compare it with the case where the DDI strategy with the SMA method is adopted.

3.4. Bullwhip effect

In this subsection, we are interested in studying the bullwhip effect occurring in the considered supply chain. Let $\tilde{\psi}_j$, $\tilde{\tilde{\psi}}_j$ and $\tilde{\tilde{\tilde{\psi}}}_j$ be the infinite moving average representation (IMAR) coefficients of the orders processes in the cases where the WMA, SMA and MMSE methods are adopted for the demand forecasts, respectively.

First, when the WMA method is used in order to forecast the customer's demand, the $ARMA(p,q)$ demand process at the retailer where ξ_t is the error term that transforms into an $ARMA(p,q)$ order process at the manufacturer, where $\breve{\xi}_t = L[\sum_{i=1}^{N} x_i(\xi_{t-i-1} - \xi_{t-i}) + \xi_t]$ is the error term. Considering the lead-time L, the parameter N and the IMAR coefficients ψ_j and $\tilde{\psi}_j$ of the demand and order processes, respectively, the bullwhip effect is measured by Eq. (6) as follows:

$$\begin{aligned} \boldsymbol{BWeffect^{WMA}}(\boldsymbol{x}) &= \frac{\boldsymbol{Var(Y_t)}}{\boldsymbol{Var(D_t)}} \\ &= \left[L^2\left(x_1^2 + x_N^2 + \sum_{i=1}^{N-1}(x_{i+1}-x_i)^2\right) + 2Lx_1 + 1\right]\left(\frac{\sum_{j=0}^{+\infty}\tilde{\psi}_j^2}{\sum_{j=0}^{+\infty}\psi_j{}^2}\right) \end{aligned} \tag{6}$$

Second, when the SMA forecasting method is adopted, the $ARMA(p,q)$ demand process for the retailer where ξ_t is the error term transforms into an $ARMA(p,q)$ order process at the manufacturer, where $\tilde{\tilde{\xi}}_t = (\frac{L}{N}+1)\xi_t - \frac{L}{N}\xi_{t-N}$ is the error term (Tliche et al., 2019). Considering the lead-time L, the parameter N and the IMAR coefficients ψ_j and $\tilde{\tilde{\psi}}_j$ of the demand and order processes, respectively, the bullwhip effect is measured by equation (7) as follows:

$$BWeffect^{SMA} = \frac{Var(Y_t)}{Var(D_t)} = \frac{2L^2+N^2+2NL}{N^2}\left(\frac{\sum_{j=0}^{+\infty}\tilde{\tilde{\psi}}_j{}^2}{\sum_{j=0}^{+\infty}\psi_j{}^2}\right) \tag{7}$$

Third, when the MMSE forecasting method is adopted by the retailer, the $ARMA(p,q)$ process at the retailer transforms into an $ARMA(p, Max(p, q-L))$ process at the producer (Zhang, 2004). Considering the IMAR coefficients of demand and orders processes, respectively, ψ_j and $\tilde{\tilde{\tilde{\psi}}}_j$, the ratio of the unconditional variance of the orders process to that of demand process, namely the Bullwhip effect is measured by equation (8) as follows:

$$BWeffect^{MMSE} = \frac{Var(Y_t)}{Var(D_t)} = \left(\sum_{j=0}^{L}\psi_j\right)^2\left(\frac{\sum_{j=0}^{+\infty}\tilde{\tilde{\tilde{\psi}}}_j{}^2}{\sum_{j=0}^{+\infty}\psi_j{}^2}\right) \tag{8}$$

Furthermore, considering the obtained expressions of Eqs. (6), (7) and (8), we simply consider the ratio of $BWeffect^{WMA}$ to $BWeffect^{MMSE}$ and the ratio $BWeffect^{WMA}$ to $BWeffect^{SMA}$. In this manner, we obtain ideas about how the bullwhip effect behaves when switching from a NIS strategy with the MMSE method to a DDI strategy with the WMA method (*case 1*), and when switching from a DDI strategy with the SMA method to a DDI strategy with the WMA method (*case 2*). Let denote the bullwhip effect evolution of the *case 1* by BEE^{WMA}_{MMSE}, which is expressed by Eq. (9) as follows:

$$\begin{aligned} BEE^{WMA}_{MMSE} &= \frac{BWeffect^{WMA}(x)}{BWeffect^{MMSE}} \\ &= \frac{\left[L^2\left(x_1^2+x_N^2+\sum_{i=1}^{N-1}(x_{i+1}-x_i)^2\right)+2Lx_1+1\right]}{\left(\sum_{j=0}^{L}\psi_j\right)^2}\left(\frac{\sum_{j=0}^{+\infty}\tilde{\psi}_j^2}{\sum_{j=0}^{+\infty}\tilde{\tilde{\tilde{\psi}}}_j{}^2}\right) \end{aligned} \tag{9}$$

Now, let denote the bullwhip effect evolution of the *case 2* by BEE^{WMA}_{SMA}, which is expressed by Eq. (10a) as follows:

$$BEE^{WMA}_{SMA} = \frac{BWeffect^{WMA}}{BWeffect^{SMA}}$$

$$= \frac{N^2\left[L^2\left(x_1^2+x_N^2+\sum_{i=1}^{N-1}(x_{i+1}-x_i)^2\right)+2Lx_1+1\right]}{2L^2+N^2+2NL}\left(\frac{\sum_{j=0}^{+\infty}\tilde{\psi}_j^2}{\sum_{j=0}^{+\infty}\tilde{\tilde{\psi}}_j^2}\right) \tag{10a}$$

We note here that $\tilde{\psi}_j$ are equal to $\tilde{\tilde{\psi}}_j$ for j from 0 to $+\infty$ since the order processes Y_t keep the same coefficients ϕ_j and θ_j of the demand processes in the cases where WMA and SMA methods are adopted, respectively. Indeed, the only difference between the two structures of Y_t is in the error terms. Hence, equation (10a) is equivalent to:

$$BEE_{SMA}^{WMA} = \frac{BWeffect^{WMA}}{BWeffect^{SMA}} = \frac{N^2\left[L^2\left(x_1^2+x_N^2+\sum_{i=1}^{N-1}(x_{i+1}-x_i)^2\right)+2Lx_1+1\right]}{2L^2+N^2+2NL} \tag{10}$$

The mathematical expression of Eq. (10) is not a linear function. Studying this equation is not a straightforward task since it does not allow one to understand the domains in which BEE_{SMA}^{WMA} is inferior or superior to 1. Therefore, we suggest some simulations for this metric in Section 4 to have an approximate idea of the gap of the bullwhip effect, such as separating the situations in which WMA and SMA are adopted. Note that BEE_{SMA}^{WMA} will be noted by $BEE_{SMA}^{WMA/Newton}$ since the vector x in the simulation section is the Newton's optimal weighting.

Once the analytical expressions for the different supply chain performance metrics are derived, we proceed to detail the problem model and the resolution method.

3.5. Newton method for optimal weighting

We assume that the manufacturer aims to minimize his average inventory level when forecasting over the time period $L+1$. In this work, this inventory-oriented enhancement is the main engine of the supply chain surplus. Indeed, if the possibility of inventory level minimization still exists, then the value of this gap is convertible to a monetary value that can be shared across the supply chain. To do this, since the inventory expression in equation (5) is a function of the MSE, the manufacturer is recommended to simply determine the weighting vector x^* that minimizes this MSE. Then, the expression of the $MSE^{DDI}(x)$ in Eq. (4) is replaced by the obtained value $MSE^{DDI}(x^*)$, and the optimal average inventory level $\bar{I}_t^{DDI}(x^*)$ is then determined. Let us first define the MFOP, which can be expressed as follows:

$$(MFOP):\left\{\begin{array}{l}\textbf{minimize } MSE^{DDI}(x)\\ \text{subject to}\left\{\begin{array}{c}\sum_{i=1}^{N}x_i=1\\ x_i\ge 0\ \forall i\in\{1,\ldots,N\}\\ x=\begin{pmatrix}x_1\\ \vdots\\ x_N\end{pmatrix}\end{array}\right.\end{array}\right.$$

There are several methods that can be applied to solve such problems. In this paper, we select the Newton's method, a gradient-based iterative optimization algorithm that is widely used in the literature because of its ease of implementation and quickness of resolution since the convergence is quadratic (Qi & Sun, 1999). For a positive quadratic convex function defined on $\mathbb{R}$, the minimum is reached if the derivative function is equal to 0 and the second derivative function is positive on $\mathbb{R}$. We then talk about the 1st order and 2nd order optimality conditions. The Newton's method is suitable for the optimization in this setting because of the mathematical nature of the MSE. Indeed, the MSE is positive and of quadratic convex nature defined on $\mathbb{R}^n$. The 1st optimality conditions (considering the constraints of the weights) are presented by the KKT formulation shown next. The 2nd optimality conditions (always considering the constraints of the weights) are presented by the Hessian matrix $\nabla g\binom{x^k}{\lambda^k}$ also shown next, which must be a semi-definite positive matrix. This statement is not evident because of the complexity of the matrix components. However, a matrix is semi-definite positive if and only if all of its eigenvalues are non-negative (Vandenberghe & Boyd, 1996). In practice, we verified this condition in our simulations.

For the purpose, we go on to state our resolution methodology. We first modify the constraints' form of the MFOP into a matrix form and then the problem is rewritten as follows:

$$(MFOP):\left\{\begin{array}{l}\textbf{minimize } MSE^{DDI}(x)\\ s.t.\left\{\begin{array}{c}x\ e^t\le 1\\ -x\ e^t\le -1\\ -x_i\le 0\quad\forall i\in\{1,\ldots,N\}\\ e=\begin{pmatrix}1\\ \vdots\\ 1\end{pmatrix}_{[N,1]}\\ x=\begin{pmatrix}x_1\\ \vdots\\ x_N\end{pmatrix}\end{array}\right.\end{array}\right.$$

$$\Leftrightarrow (MFOP):\left\{\begin{array}{l}\textbf{minimize } MSE^{DDI}(x)\\ s.t.\left\{\begin{array}{l}A\ x\le b,\quad A=\begin{pmatrix}e^t\\ -e^t\\ -I_N\end{pmatrix}_{[N+2,N]}\quad\text{and}\\ b=\begin{pmatrix}1\\ -1\\ 0_N\end{pmatrix}_{[N+2,1]}\\ e=\begin{pmatrix}1\\ \vdots\\ 1\end{pmatrix}_{[N,1]},\ 0_N=\begin{pmatrix}0\\ \vdots\\ 0\end{pmatrix}_{[N,1]},\\ I_N=\begin{pmatrix}1&0&0\\ 0&\ddots&0\\ 0&0&1\end{pmatrix}_{[N,N]}\\ x=\begin{pmatrix}x_1\\ \vdots\\ x_N\end{pmatrix}\end{array}\right.\end{array}\right.$$

We first define the necessary Karush-Kuhn-Tucker (KKT) first order optimality conditions that are associated with this problem as follows (Gordon & Tibshirani, 2012):

$$(KKT):\left\{\begin{array}{l}(1):\ \frac{\partial MSE^{DDI}(x)}{\partial x}+A^t\lambda=0_N\\ (2):\ \lambda_i(Ax-b)_i=0,\ i\in\{1,\ldots,r\}\\ s.t.\left\{\begin{array}{c}Ax\le b\\ \lambda_i\ge 0,\ i\in\{1,\ldots,r\}\\ r=N+2\\ x=\begin{pmatrix}x_1\\ \vdots\\ x_N\end{pmatrix}\\ \lambda=\begin{pmatrix}\lambda_1\\ \vdots\\ \lambda_r\end{pmatrix}\end{array}\right.\end{array}\right.$$

We note here that λ_i is simply a parameter of the constraints' satisfaction and is not related to our analysis. Next, in order to derive $\frac{\partial MSE^{DDI}(x)}{\partial x}$, the $MSE^{DDI}(x)$ function must be rearranged in the following form:

$$MSE^{DDI}(x) = (L+1)^2\gamma_0\sum_{i=1}^{N}x_i^2 + 2(L+1)^2\sum_{j=1}^{N-1}\left(x_j\sum_{i=j+1}^{N}x_i\gamma_{i-j}\right) - 2(L+1)\sum_{i=1}^{L+1}\sum_{j=1}^{N}x_j\gamma_{i+j-1} + (L+1)\gamma_0 + 2\sum_{i=1}^{L} i\ \gamma_{L+1-i}$$

We separate the four components of the rearranged $MSE^{DDI}(x)$ expression, and we denote them as follows:

$$C_1(x) = (L+1)^2\gamma_0\sum_{i=1}^{N}x_i^2 = (L+1)^2\gamma_0 x I_N x^t$$

$$C_2(x) = 2(L+1)^2\sum_{j=1}^{N-1}\left(x_j\sum_{i=j+1}^{N}x_i\gamma_{i-j}\right)$$

$$C_3(x) = -2(L+1)\sum_{i=1}^{L+1}\sum_{j=1}^{N}x_j\gamma_{i+j-1} = -2(L+1)\left(\sum_{i=1}^{L+1}\mathrm{Y}_i^t\right)x$$

where $\mathrm{Y}_i = \begin{bmatrix}\gamma_i\\ \vdots\\ \vdots\\ \gamma_{i+N-1}\end{bmatrix}$, $i = 1,\ldots,L+1$

$$C_4(x) = (L+1)\gamma_0 + 2\sum_{i=1}^{L} i\ \gamma_{L+1-i}$$

We then derive the derivative functions of the four $MSE^{DDI}(x)$ components as follows:

$$\frac{\partial C_1(x)}{\partial x} = 2(L+1)^2\ \gamma_0\ I_N\ x,$$

$$\frac{\partial C_2(x)}{\partial x} = 2(L+1)^2\begin{bmatrix}\sum\limits_{\substack{i=1\\ i\neq 1}}^{N} x_i\gamma_{|i-1|}\\ \vdots\\ \vdots\\ \sum\limits_{\substack{i=1\\ i\neq N}}^{N} x_i\gamma_{|i-N|}\end{bmatrix},$$

$$\frac{\partial C_3(x)}{\partial x} = -2(L+1)\mathrm{Y}^t$$

where

$$\mathrm{Y} = \begin{bmatrix}\sum\limits_{i=1}^{L+1}\ \gamma_i\\ \vdots\\ \vdots\\ \sum\limits_{i=N}^{L+N}\ \gamma_i\end{bmatrix},$$

and

$$\frac{\partial C_4(x)}{\partial x} = 0_N.$$

Then, the derivative function of $MSE^{DDI}(x)$ is finally expressed as follows:

$$\frac{\partial MSE^{DDI}(x)}{\partial x} = \begin{bmatrix}2(L+1)^2\left(\gamma_0 x_1 + \sum\limits_{\substack{i=1\\ i\neq 1}}^{N} x_i\gamma_{|i-1|}\right) - 2(L+1)\sum\limits_{i=1}^{L+1}\ \gamma_i\\ \vdots\\ \vdots\\ \vdots\\ 2(L+1)^2\left(\gamma_0 x_N + \sum\limits_{\substack{i=1\\ i\neq N}}^{N} x_i\gamma_{|i-N|}\right) - 2(L+1)\sum\limits_{i=N}^{L+N}\ \gamma_i\end{bmatrix}_{[N,1]}$$

In a second step, we denote $G(x) = \frac{\partial MSE^{DDI}(x)}{\partial x} + A^t\lambda$, and then KKT is equivalent to the following nonlinear equations system (NLS):

$$(NLS):\begin{cases} G(x,\ \lambda) = 0_N\\ \lambda_i(b-Ax)_i = 0,\ \lambda_i \geq 0,\ (b-Ax)_i \geq 0,\ i\in\{1,\ldots,r\}\end{cases}$$

Proposition (Chen, Chen & Kanzow, 2000a):

If $a \geq 0$ and $b \geq 0$, then $ab = 0 \Leftrightarrow \varphi(a,b) = 0$ with $\varphi(a,b) = a + b - \sqrt{a^2+b^2}$.

By applying the *Proposition* to (NLS), we obtain the following system (NLS'):

$$(NLS'):\begin{cases} G(x,\ \lambda) = 0_N\\ \psi_i(\lambda_i,(b-Ax)_i) = \lambda_i + (b-Ax)_i - \sqrt{\lambda_i^2 + (b-Ax)_i^2}\\ \quad = 0,\ i\in\{1,\ldots,r\}\end{cases}$$

Next, let $g\binom{x}{\lambda} = \binom{G(x,\ \lambda)}{\psi(x,\ \lambda)} = 0_{N+r}$. Consequently, solving (NLS') requires solving the following nonlinear equation (S):

$$(S):\ \begin{pmatrix}x^{k+1}\\ \lambda^{k+1}\end{pmatrix} = \begin{pmatrix}x^k\\ \lambda^k\end{pmatrix} - \nabla g^{-1}\begin{pmatrix}x^k\\ \lambda^k\end{pmatrix}.g\begin{pmatrix}x^k\\ \lambda^k\end{pmatrix}$$

The resolution of such an equation requires the computation of the inverse of the Hessian matrix $\nabla g^{-1}\binom{x^k}{\lambda^k}$ at each iteration k, which could be expensive in terms of time and memory. The best solution is then to solve a linear system using the Pivot-Gauss method (Sorensen, 1985). For any linear system of the form $Ax = b$, the Pivot-Gauss method consists of staggering the system by making changes to the rows of matrix A of the type $L_i \leftarrow L_i + \alpha L_j$ to obtain the solution at the end a triangular matrix. Finally, solving (S) amounts to solving the following linear equations system (LS):

$$(LS):\ \nabla g\begin{pmatrix}x^k\\ \lambda^k\end{pmatrix}.\Delta u^k = -g\begin{pmatrix}x^k\\ \lambda^k\end{pmatrix}$$

where $\Delta u^k = \binom{\Delta x^k}{\Delta\lambda^k} = \binom{x^{k+1}-x^k}{\lambda^{k+1}-\lambda^k}$ and

$$\nabla g\begin{pmatrix}x^k\\ \lambda^k\end{pmatrix} = \begin{bmatrix}\frac{\partial G(x^k,\lambda^k)}{\partial x} & \frac{\partial G(x^k,\lambda^k)}{\partial\lambda}\\ \frac{\partial\psi(x^k,\lambda^k)}{\partial x} & \frac{\partial\psi(x^k,\lambda^k)}{\partial\lambda}\end{bmatrix}_{[N+r,\ N+r]}$$

with

$$\frac{\partial G(x^k,\lambda^k)}{\partial x} = \frac{\partial^2 MSE^{DDI}(x^k)}{\partial x^2} = \begin{bmatrix}2(L+1)^2\gamma_0 & \cdots & 2(L+1)^2\gamma_{|1-N|}\\ \vdots & \ddots & \vdots\\ 2(L+1)^2\gamma_{|N-1|} & \cdots & 2(L+1)^2\gamma_0\end{bmatrix},$$

$$\frac{\partial G(x^k,\lambda^k)}{\partial\lambda} = A^t,$$

$$\frac{\partial \psi\left(x^k,\lambda^k\right)}{\partial x} = -A + \frac{\left(b - Ax^k\right)_i A}{\sqrt{\left(\lambda_i^k\right)^2 + \left(b - Ax^k\right)_i^2}}$$

$$\frac{\partial \psi\left(x^k,\lambda^k\right)}{\partial \lambda} = \left(1 - \frac{\lambda_i}{\left(\lambda_i^k\right)^2 + \left(b - Ax^k\right)_i^2}\right) I_{(r,r)}$$

In this section, we established the transformation of the quadratic problem MFOP into a system of linear equations (LS). The use of the Pivot-Gauss method allows for the reduction of the execution time that is necessary to obtain the Newton's results.

We conclude this section by summarizing the collaborative process in the considered supply chain. The manufacturer signs an income-sharing contract with the retailer. The latter agrees to adopt the WMA/Newton method in his demand forecasting, thus allowing the demand inference at the manufacturer. The manufacturer implements the Newton's method to obtain the optimal allocation vector according to his average inventory level. Once the system (LS) is solved, the manufacturer passes the information on the allocation vector to the retailer who will implement this weighting in his WMA forecasting method. The reduction of the MSE and consequently the average inventory level at the manufacturer, generates savings that will be shared with the retailer.

4. Simulation results and discussion

In this section, we carry out some simulated experiments of our implementation, namely, the resolution of some examples that will serve to validate the approach. Then, we discuss the observed results compared to the NIS strategy with the MMSE method and compared to the DDI strategy with the SMA method.

4.1. Implementation of newton's method

Using the MATLAB software, we implemented the Newton's method for solving a quadratic problem under linear constraints. We adapted the general form of the quadratic problem to coincide with our MFOP and then conducted simulations by solving some problems using different predefined demand processes. The pseudocode of the Newton's algorithm is shown as follows:

Newton's algorithm:

- ➢ Input of the problem data N, L, A, b and γ_i for $i \in \{1,\ldots,N+L\}$
- ➢ Input of the algorithm parameters:
  i_{max} (maximal iterations number) and ε (maximal accepted error)
- ➢ Entry of the initial estimates: $k=0$, $x^0 = \begin{pmatrix} x_1^0 \\ : \\ x_N^0 \end{pmatrix}$ and $\lambda^0 = \begin{pmatrix} \lambda_1^0 \\ : \\ \lambda_r^0 \end{pmatrix}$
- ➢ While $g\begin{pmatrix} x^k \\ \lambda^k \end{pmatrix} \geq \varepsilon$ or $\begin{matrix} x^{k+1} - x^k \\ \lambda^{k+1} - \lambda^k \end{matrix} \geq \varepsilon$ or $k < i_{max}$
  - ■ Compute $g\begin{pmatrix} x^k \\ \lambda^k \end{pmatrix}$ and $\nabla g\begin{pmatrix} x^k \\ \lambda^k \end{pmatrix}$
  - ■ Solve $\nabla g\begin{pmatrix} x^k \\ \lambda^k \end{pmatrix}.\Delta u^k = -g\begin{pmatrix} x^k \\ \lambda^k \end{pmatrix}$ using the Pivot-Gauss method and deduce
    $\begin{pmatrix} x^{k+1} \\ \lambda^{k+1} \end{pmatrix} = \begin{pmatrix} x^k \\ \lambda^k \end{pmatrix} + \Delta u^k = \begin{pmatrix} x^k \\ \lambda^k \end{pmatrix} + \begin{pmatrix} \Delta x^k \\ \Delta \lambda^k \end{pmatrix}$
  - ■ $k = k+1$

  End
- ➢ If $k = i_{max}$, then the algorithm diverges and it will be necessary to change the initial point x^0.

4.2. Simulation experiments

In this first part of the simulation, we consider the demand models in, which are causal invertible $ARMA(p,q)$ models, and we vary the autoregressive parameters ϕ_j where $j = 1,\ldots,p$ and the moving average parameters θ_j where $j = 1,\ldots,q$. Since it is impossible to infinitely compute the IMAR coefficients, we only compute the first 1000 ψ-weights for all simulated $ARMA(p,q)$ demand processes. Then, we conduct comparative studies between the cases where NIS with MMSE and DDI with WMA/Newton, and a comparative study between the cases where DDI with WMA/Newton method and DDI with SMA method, for the following fixed parameters: $c = 10$, $\sigma_\xi^2 = 1$, $L = 5$, $N = 12$, $h = 1$, and $s = 2$. The optimal ponderation vector is obtained by applying the Newton's method. The chosen parameters of Newton's algorithm are as follows: $\varepsilon = 10^{-5}$ and $i_{max} = 100$. The initial solution can be arbitrarily chosen as long as it is in the realm of feasible solutions. The eigenvalues of the Hessian matrix are positive and for different initial solutions corresponding to multiple simulations on the same problem, the optimal solution is always unique. This ensures the global optimality of the Newton's solution. Finally, the Newton's algorithm does not exceed a dozen iterations and the elapsed time is on the scale of a second using the Windows 7 professional operating system.

4.3. Comparative studies

The following tables present the findings of our simulations on 20 different demand models. We selected 20 different demand models used in the simulation for the simple reason of multiple illustrations, where we variate autoregressive and moving average parameters of the demand processes. Multiple simulations procure more credibility about the robustness of results. Table 2 shows the coefficients of the demand processes and the obtained Newton's optimal weights for the N past observations. Table 3 shows the simulation results of the MSE and $\tilde{I}_t$, respectively, when the NIS strategy is adopted, when the DDI strategy with the SMA method is adopted, and finally when the DDI strategy with the WMA/Newton method is adopted.

Table 3 reports two important results. The first one is that this table exhibits the effectiveness of the DDI strategy with WMA/Newton compared to the NIS approach. Therefore, the DDI strategy remains valuable when there is no information sharing mechanisms, regardless of the used forecasting method. Besides, based on simulated models in Table 3, the WMA method with the Newton's allocation proves its efficiency by outperforming the SMA method with regards to the two performance metrics. It's about the second result where this table proves that decision-makers in supply chains can enhance their DDI performance and market competitiveness by simply considering the optimal weighting that is generated by Newton's method, rather than considering an equitable weighting of the order of $1/N$. As expected, the enhancement of the two metrics is different when we vary the autoregressive and the moving average parameters of the demand processes. This is due to the nonlinear relation mentioned above in equation (5) that connects the forecast MSE to its effective consequence, the average inventory level.

Besides, since there are no specific "standard approaches" for determining the best configuration, and for investigation purposes, we study in the next subsection the sensibility of these metrics according to the lead-time L and moving average parameter N values. For illustration purposes, we consider an arbitrary example of an $ARMA(3,2)$ demand process, which is defined as follows:

$$D_t = 10 + 0.6\ D_{t-1} + 0.4\ D_{t-1} - 0.3\ D_{t-1} + \xi_t + 0.1\ \xi_{t-1} + 0.08\ \xi_{t-2}$$

4.3.1. Comparison between the ddi strategy with WMA/Newton method and the nis with mmse method with respect to lead time and moving average parameters

Based on the comparison between DDI with WMA/Newton results and NIS with MMSE results, we study the sensibilities of the

Table 2
Optimal Newton's weights for $ARMA(p,q)$ demand models .

Demand model	Autoregressive and moving average coefficients	Newton weights vector x^*
1	$\phi_1 = 0.400$	$\begin{pmatrix}0.2218; 0.0667; 0.0667; 0.0667; 0.0667; 0.0667;\\ 0.0667; 0.0667; 0.0667; 0.0667; 0.0667; 0.1112\end{pmatrix}$
2	$\phi_1 = 0.500$	$\begin{pmatrix}0.2835; 0.0597; 0.0597; 0.0597; 0.0597; 0.0597;\\ 0.0597; 0.0597; 0.0597; 0.0597; 0.0597; 0.1194\end{pmatrix}$
3	$\phi_1 = 0.600$	$\begin{pmatrix}0.3653; 0.0508; 0.0508; 0.0508; 0.0508; 0.0508;\\ 0.0508; 0.0508; 0.0508; 0.0508; 0.0508; 0.1269\end{pmatrix}$
4	$\theta_1 = 0.400$	$\begin{pmatrix}0.1727; 0.0370; 0.0913; 0.0696; 0.0782; 0.0749;\\ 0.0758; 0.0764; 0.0739; 0.0806; 0.0636; 0.1061\end{pmatrix}$
5	$\theta_1 = 0.500$	$\begin{pmatrix}0.1952; 0.0143; 0.1046; 0.0596; 0.0818; 0.0714;\\ 0.0753; 0.0760; 0.0704; 0.0837; 0.0560; 0.1118\end{pmatrix}$
6	$\theta_1 = 0.600$	$\begin{pmatrix}0.2116; 0.0000; 0.1171; 0.0467; 0.0880; 0.0659;\\ 0.0756; 0.0756; 0.0659; 0.0880; 0.0476; 0.1171\end{pmatrix}$
7	$\phi_1 = 0.400$ $\theta_1 = 0.051$	$\begin{pmatrix}0.2397; 0.0567; 0.0661; 0.0656; 0.0656; 0.0656;\\ 0.0656; 0.0656; 0.0656; 0.0657; 0.0631; 0.1149\end{pmatrix}$
8	$\phi_1 = 0.400$ $\theta_1 = 0.100$	$\begin{pmatrix}0.2569; 0.0455; 0.0666; 0.0645; 0.0647; 0.0647;\\ 0.0647; 0.0647; 0.0646; 0.0652; 0.0593; 0.1186\end{pmatrix}$
9	$\phi_1 = 0.400$ $\theta_1 = 0.300$	$\begin{pmatrix}0.3186; 0.0000; 0.0752; 0.0575; 0.0628; 0.0613;\\ 0.0615; 0.0621; 0.0596; 0.0680; 0.0400; 0.1334\end{pmatrix}$
10	$\phi_1 = 0.400$ $\theta_1 = 0.300$ $\theta_2 = 0.100$	$\begin{pmatrix}0.3550; 0.0000; 0.0449; 0.0692; 0.0574; 0.0586;\\ 0.0593; 0.0585; 0.0616; 0.0565; 0.0413; 0.1378\end{pmatrix}$
11	$\phi_1 = 0.400$ $\theta_1 = 0.300$ $\theta_2 = 0.150$	$\begin{pmatrix}0.3733; 0.0000; 0.0271; 0.0758; 0.0572; 0.0557;\\ 0.0582; 0.0578; 0.0624; 0.0504; 0.0420; 0.1400\end{pmatrix}$
12	$\phi_1 = 0.400$ $\theta_1 = 0.300$ $\theta_2 = 0.200$	$\begin{pmatrix}0.3909; 0.0015; 0.0068; 0.0829; 0.0593; 0.0514;\\ 0.0567; 0.0583; 0.0631; 0.0441; 0.0428; 0.1423\end{pmatrix}$
13	$\phi_1 = 0.400$ $\theta_1 = 0.300$ $\theta_2 = 0.180$ $\theta_3 = 0.060$ $\theta_4 = 0.050$	$\begin{pmatrix}0.4138; 0.0091; 0.0259; 0.0590; 0.0410; 0.0602;\\ 0.0562; 0.0493; 0.0551; 0.0464; 0.0425; 0.1415\end{pmatrix}$
14	$\phi_1 = 0.200$ $\phi_2 = 0.150$ $\theta_1 = 0.100$	$\begin{pmatrix}0.2194; 0.1045; 0.0584; 0.0630; 0.0626; 0.0626;\\ 0.0626; 0.0626; 0.0627; 0.0614; 0.0742; 0.1059\end{pmatrix}$
15	$\phi_1 = 0.200$ $\phi_2 = 0.150$ $\phi_3 = 0.120$ $\phi_4 = 0.100$ $\theta_1 = 0.100$	$\begin{pmatrix}0.2806; 0.1607; 0.1142; 0.0694; 0.0308; 0.0347;\\ 0.0344; 0.0336; 0.0415; 0.0509; 0.0615; 0.0878\end{pmatrix}$
16	$\phi_1 = 0.200$ $\phi_2 = 0.150$ $\phi_3 = 0.120$ $\phi_4 = 0.100$ $\theta_1 = 0.100$ $\theta_2 = 0.065$	$\begin{pmatrix}0.3018; 0.1767; 0.0979; 0.0623; 0.0261; 0.0321;\\ 0.0335; 0.0318; 0.0389; 0.0463; 0.0629; 0.0899\end{pmatrix}$
17	$\phi_1 = 0.200$ $\phi_2 = 0.150$ $\phi_3 = 0.120$ $\phi_4 = 0.100$ $\theta_1 = 0.100$ $\theta_2 = 0.065$ $\theta_3 = 0.060$ $\theta_4 = 0.051$	$\begin{pmatrix}0.3293; 0.1994; 0.1171; 0.0501; 0.0000; 0.0177;\\ 0.0256; 0.0257; 0.0357; 0.0468; 0.0629; 0.0898\end{pmatrix}$
18	$\phi_1 = 0.200$ $\phi_2 = -0.150$ $\phi_3 = 0.120$ $\phi_4 = -0.100$ $\phi_5 = 0.080$ $\phi_6 = 0.070$ $\phi_7 = 0.060$ $\phi_8 = -0.051$ $\theta_1 = 0.100$	$\begin{pmatrix}0.1484; 0.0679; 0.1154; 0.0721; 0.1023; 0.0792;\\ 0.0672; 0.0589; 0.0657; 0.0760; 0.0606; 0.0865\end{pmatrix}$

(*continued on next page*)

Table 2 (*continued*)

Demand model	Autoregressive and moving average coefficients	Newton weights vector x^*
19	$\phi_1 = 0.200$ $\phi_2 = -0.150$ $\phi_3 = 0.120$ $\phi_4 = -0.100$ $\phi_5 = 0.080$ $\phi_6 = 0.070$ $\phi_7 = 0.060$ $\phi_8 = -0.051$ $\theta_1 = 0.100$ $\theta_2 = 0.060$	$\begin{pmatrix} 0.1614;\ 0.0772;\ 0.1089;\ 0.0709;\ 0.0992;\ 0.0770; \\ 0.0627;\ 0.0550;\ 0.0642;\ 0.0734;\ 0.0617;\ 0.0884 \end{pmatrix}$
20	$\phi_1 = 0.200$ $\phi_2 = -0.150$ $\phi_3 = 0.120$ $\phi_4 = -0.100$ $\phi_5 = 0.080$ $\phi_6 = 0.070$ $\phi_7 = 0.060$ $\phi_8 = -0.051$ $\theta_1 = 0.100$ $\theta_2 = 0.060$ $\theta_3 = 0.040$ $\theta_4 = 0.010$	$\begin{pmatrix} 0.1712;\ 0.0842;\ 0.1169;\ 0.0648;\ 0.0958;\ 0.0730; \\ 0.0592;\ 0.0499;\ 0.0600;\ 0.0737;\ 0.0627;\ 0.0887 \end{pmatrix}$

Table 3
SE and $\tilde{I}_t$ results for $ARMA(p, q)$ demands when NIS, DDI with SMA and DDI with WMA/Newton methods, are adopted.

Demand Model	NIS with MMSE method		DDI with SMA method		DDI with WMA/Newton method	
	MSE^{NIS}	$\tilde{I}_t^{NIS}$	MSE^{DDI}	$\tilde{I}_t^{DDI}$	MSE^{DDI*}	$\tilde{I}_t^{DDI*}$
1	67.3756	19.6936	20.3867	11.5614	19.4392	11.4855
2	138.3996	33.0452	26.7926	14.3894	24.6456	14.2098
3	300.1029	62.3794	36.5808	18.7090	31.8348	18.2922
4	11.7600	7.8951	16.2400	07.4301	15.8909	7.4038
5	13.5000	8.5608	18.5000	07.7789	17.9541	7.7376
6	15.3600	9.3215	20.9400	08,1536	20.1583	8.0942
7	74.3786	21.3331	22.3074	11.8812	21.0996	11.7838
8	81.4330	23.0569	24.2527	12.2041	22.7645	12.0835
9	113.5408	31.7447	33.2078	13.6816	30.2764	13.4401
10	131.3007	37.0936	37.4282	14.4392	33.4410	14.1049
11	140.6645	40.1215	39.6913	14.8415	35.1010	14.4536
12	150.3507	43.2234	42.0563	15.2594	36.8125	14.8132
13	166.6726	48.7798	44.6284	15.8453	38.2068	15.2839
14	49.8496	17.7593	19.6140	10.8756	18.5583	10.7887
15	82.8049	34.5483	24.1279	15.9735	20.8552	15.6680
16	90.6387	37.3469	26.5067	16.4101	22.4153	16.0256
17	99.4028	41.1695	29.7530	17.0342	23.9784	16.4813
18	14.6534	10.3757	12.8447	8.4158	12.5980	8.3972
19	15.5426	10.5792	13.8480	8.6012	13.5038	8.5747
20	16,4370	10,6580	14.4736	8.7396	14.0079	8.7030

two performance metrics with respect to the lead-time L and the moving average N. In the cases where the lead-time L is fixed and N varies, Fig. 1 presents the simulation results in terms of the MSE^{DDI} and $\tilde{I}_t^{DDI}$ improvements in percentages. These improvement percentages are computed as follows:

$$MSE^{DDI}\_Improvement = \left| \frac{MSE^{DDI*} - MSE^{NIS}}{MSE^{NIS}} \right| \times 100$$

and

$$\tilde{I}_t^{DDI}\_Improvement = \left| \frac{\tilde{I}_t^{DDI*} - \tilde{I}_t^{NIS}}{\tilde{I}_t^{NIS}} \right| \times 100$$

The obtained results in Fig. 1 show that the evolution of the improvements with respect to N is a linear function. This means that the more the parameter N increases, the more the DDI strategy with WMA/Newton is more efficient in comparison with the NIS approach. This result is expected since the parameter N does not interfere in the MMSE method used in the NIS approach. In terms of MSE and average inventories, managers are advised to increase their parameter N as well as possible while their lead-time is constant.

In the same way, Fig. 2 schematically presents the simulation results in terms of percentage improvements where the moving average parameter N is fixed and the lead-time L varies.

The same reasoning is adopted. The obtained results in Fig. 2 show that the evolution according to L is a logarithmic function. That is, for a fixed parameter N, the evolution of the enhancement in percentage becomes less important as the lead-time L becomes more important. Indeed, for low values of L, the evolution in performance is important in comparison with cases where the values of L are high. This result further confirms that the lead-time value always plays an important role in the performance of the supply chains.

4.3.2. Comparison between the ddi strategy with WMA/Newton method and the ddi strategy with sma method with respect to lead time and moving average parameters

Based on the comparison between DDI with WMA/Newton results and DDI with SMA results, we study the sensibilities of the

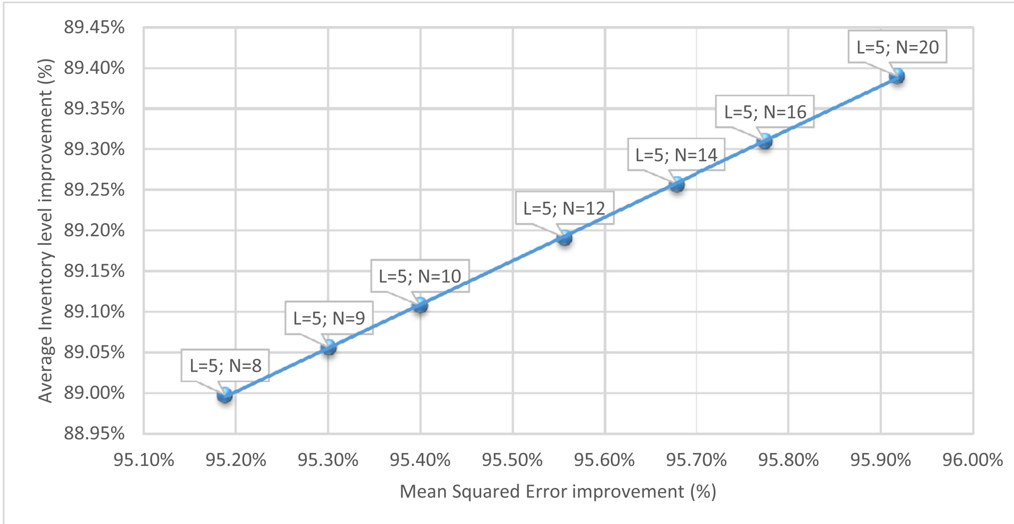

Fig. 1. Improvements of adopting DDI strategy with WMA/Newton method rather than adopting NIS strategy according to the moving average parameter *N*.

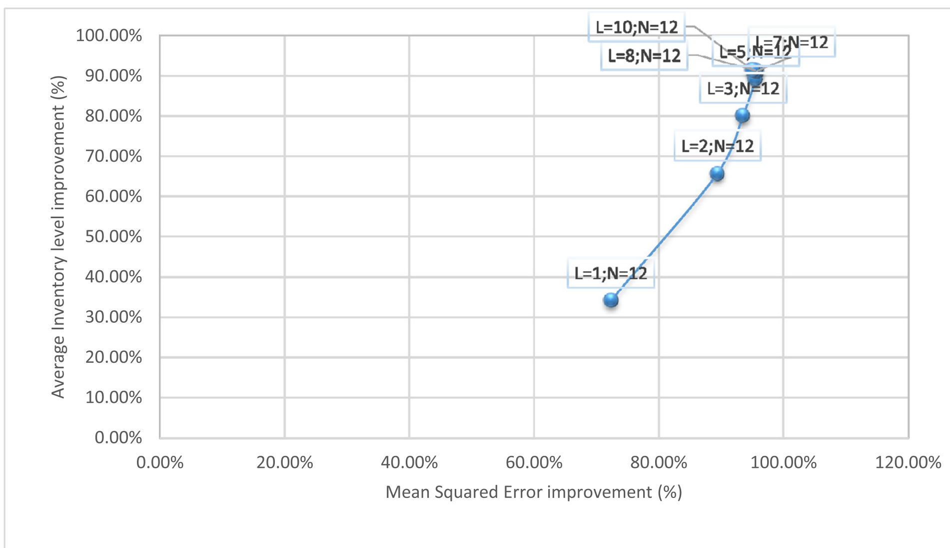

Fig. 2. Improvements of adopting DDI strategy with WMA/Newton method rather than adopting NIS strategy according to the moving average parameter *L*.

two performance metrics with respect to the lead-time L and the moving average N. In the cases where the lead-time L is fixed and N varies, Fig. 4 presents the simulation results in terms of the MSE^{DDI} and $\tilde{I}_t^{DDI}$ improvements in percentages. These improvement percentages are computed as follows:

$$MSE^{DDI}\_Improvement = \left|\frac{MSE^{DDI*} - MSE^{DDI}}{MSE^{DDI}}\right| \times 100$$

and $\tilde{I}_t^{DDI}\_Improvement = |\frac{\tilde{I}_t^{DDI*} - \tilde{I}_t^{DDI}}{\tilde{I}_t^{DDI}}| \times 100$

The obtained results in Fig. 3 show that the evolution of the improvements with respect to N is a concave function. These numerical results show that this function attains its maximal enhancement at $N = 10$ for $L = 5$. This corresponds to a 7.44% improvement in the average inventory savings. Otherwise, the enhancement is not optimal, but it still exists. In practice, the decision-makers can conduct some simulations by varying the parameter N over a fixed interval and then by choosing the value that maximizes this enhancement.

In the same way, Fig. 4 schematically presents the simulation results in terms of percentage improvements where the moving average parameter N is fixed and the lead-time L varies.

The same reasoning is adopted in Fig. 4. The obtained results show that the evolution according to L is also a concave function. These numerical results show that this function attains its maximal enhancement at $L = 3$ for $N = 12$. This corresponds to a 7.65% improvement in the average inventory savings. Generally, the lead-time value does not change since it depends on the transportation and logistics systems, and managers do not truly have the power to easily manipulate its value.

4.3.3. Evolution of the bullwhip effect

In this subsection, we study the evolution of the bullwhip effect that is associated with the WMA/Newton forecast method. Thus, we consider an example of an $ARMA(2, 2)$ demand process with the following fixed parameters set: $c = 10$, $\phi_1 = 0.4$, $\phi_2 = 0.2$, $\theta_1 = 0.15$, $\theta_2 = 0.10$ and $\sigma_\xi^2 = 1$. We mainly compute the $BEE_{MMSE}^{WMA/Newton}$ and $BEE_{SMA}^{WMA/Newton}$ indicators in Eqs. (9) and (10) in order to approximate the gap of the bullwhip effect, thereby

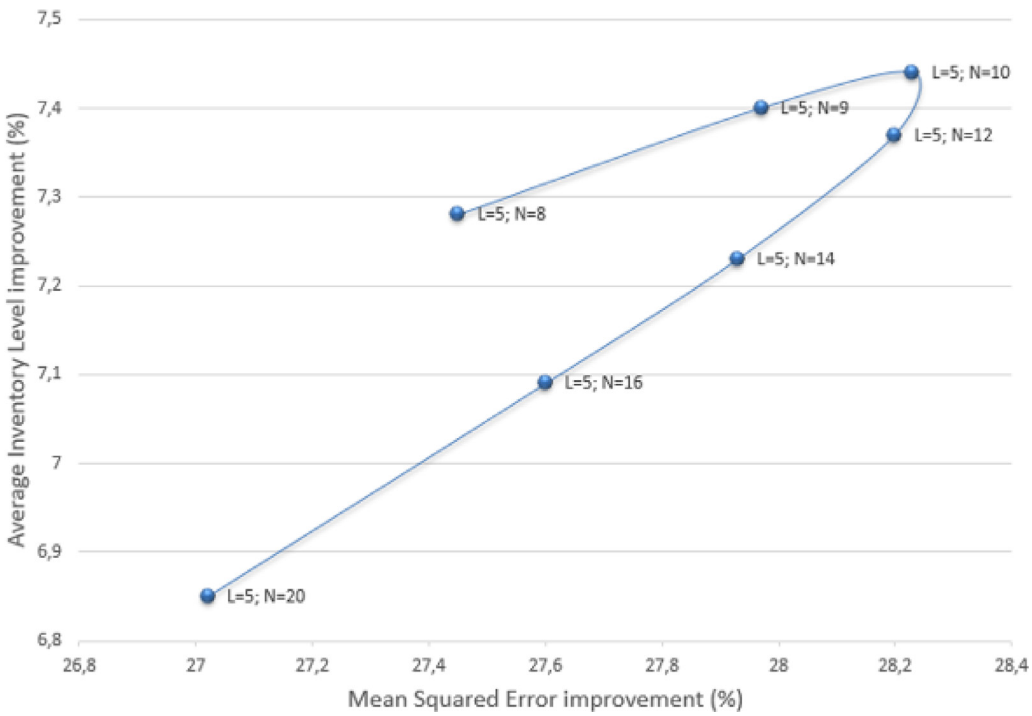

Fig. 3. Improvements of adopting DDI strategy with WMA/Newton method rather than adopting DDI with SMA according to the moving average parameter *N*.

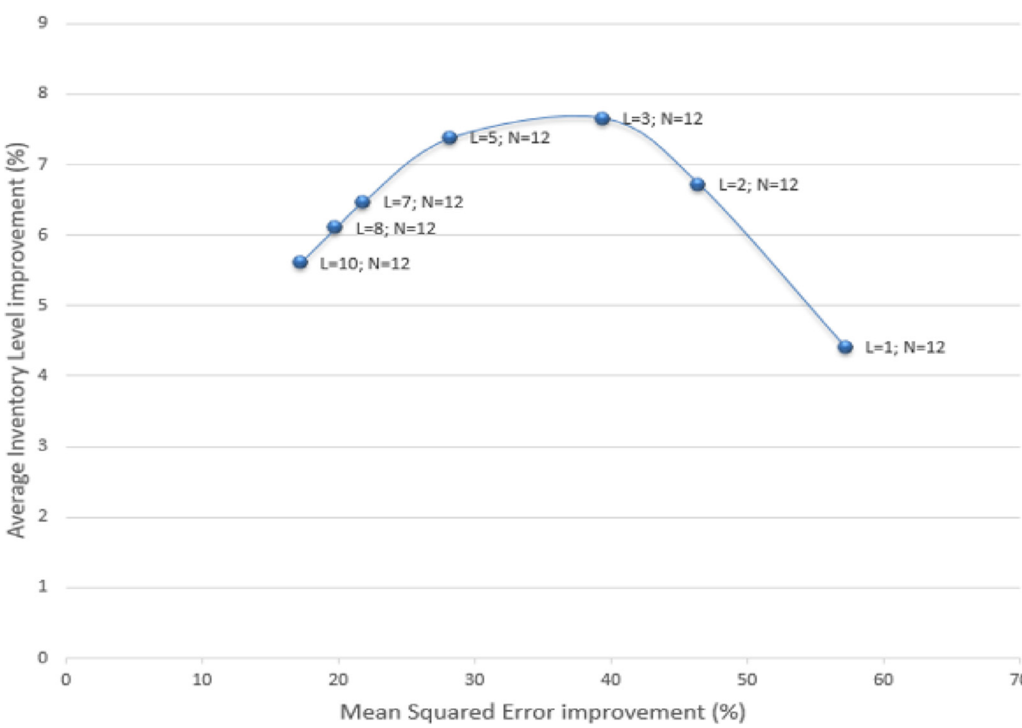

Fig. 4. Improvements of adopting DDI strategy with WMA/Newton method rather than adopting DDI with SMA according to the lead-time *L*.

separating on one hand, the DDI strategy with the WMA/Newton method to the NIS strategy with the MMSE method, and on the second hand, the DDI strategy with the WMA/Newton method to the DDI strategy with the SMA method. While these indicators are functions of the moving average N and lead-time L, we also check the variations according to these two parameters.

Fig. 5 illustrates the behavior of the evolution of the bullwhip effect when an actor switches from the MMSE method in a NIS strategy to the WMA/Newton method in a DDI strategy. For a fixed configuration of the parameter N, the performance of the WMA/Newton method becomes more important as the lead-time L decreases. In this example, for $N = 8$, DDI with WMA/Newton is valuable in terms of bullwhip effect if the lead-time value is equal to 2. Next, for a fixed lead-time L, the performance of the WMA/Newton method is more valuable as the parameter N increases. In terms of L, the results show that the value of the break-point N increases as the lead-time L increases. This is expected as generally, the performance of the MMSE method compared to the

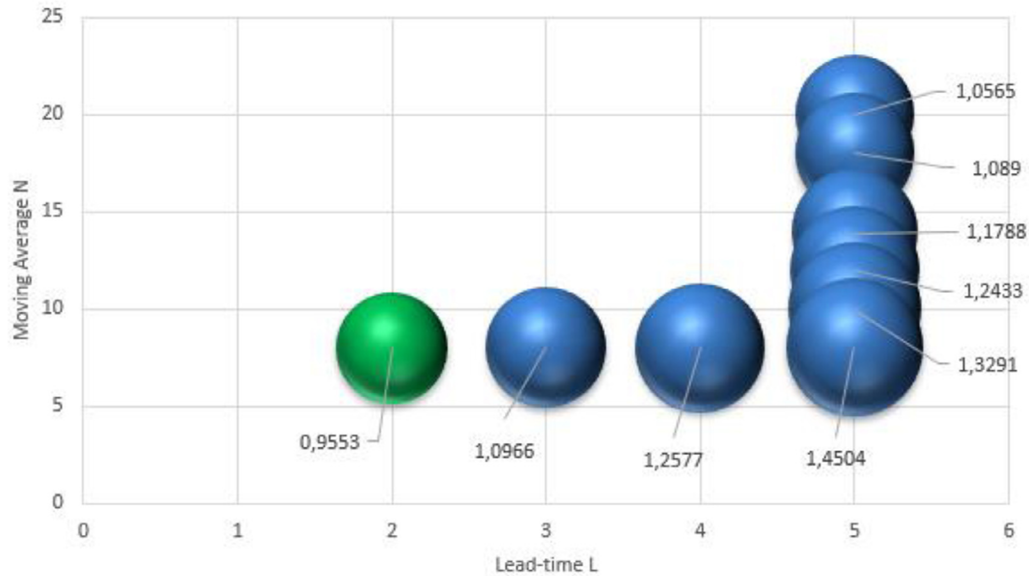

Fig. 5. Simulated $BEE_{MMSE}^{WMA/Newton}$ indicator according to moving average N and lead-time L.

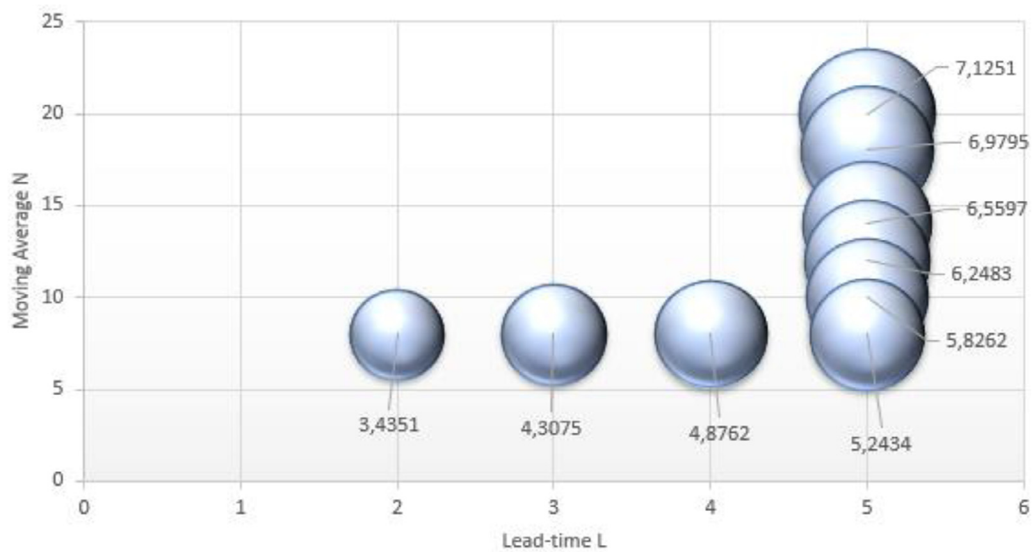

Fig. 6. Simulated $BEE_{SMA}^{WMA/Newton}$ indicator according to moving average N and lead-time L.

WMA/Newton improves with increasing the lead-time value. On the other hand, the results show that the break-point L decreases with the value of N as the performance of WMA/Newton improves with the length of the history being used. We conclude that for each value of the lead-time L, there exists a unique threshold of N from which the DDI strategy with the WMA/Newton method is more valuable than the NIS strategy with the MMSE method, in terms of bullwhip effect.

Fig. 6 illustrates the behavior of the bullwhip effect when the SMA and WMA/Newton methods are used in the forecasts. There are two important results. The first one is that the SMA method outperforms the WMA/Newton method in terms of bullwhip effect. Indeed, for the simulated data, the $BEE_{SMA}^{WMA/Newton}$ results are always greater than one. This is due to the unequal weights that are associated with the N past observations in equation (2). This can be argued to be a limitation of the WMA/Newton approach compared to the SMA method, since the SMA method provides lower variability of order processes. The second one is that the $BEE_{SMA}^{WMA/Newton}$ indicator increases with N or L. The results also show that these amplifications evolve in a quasi-logarithmic manner. That is, the increase in the indicator becomes less important as one of the two parameters increases. Hence, the bullwhip effect amplifies in the case of a DDI strategy where the downstream actor decides to switch from the use of the SMA method to the use of the WMA/Newton method. The amplified bullwhip effect is surely critical if the upstream actor doesn't use a safety stock as a buffer against orders variations. Indeed, excess inventory can result in waste, while insufficient inventory can lead to poor customer experience and lost business. Thus, the upstream actor is emphasized to use a reserve inventory in such context.

5. Discussion

In decentralized supply chains, actors often do not want to share their private information, especially in regard to the market demand. This variable is often considered as key data providing competitive power. Even when supply chain actors favor information sharing, other issues (the trust in the shared data, information leakage, high investment costs, systems compatibility, etc.) may still persist.

This work provides an initial attempt to introduce the WMA forecasting method in a decentralized supply chain in which actors favor adopting the DDI strategy. The propagation of demand processes using the WMA forecast method is unique. The introduction of the Newton optimization method allows for the quantification of the weighting of past demand observations with the purpose of minimizing the mean squared error and average inventory. The study of the improvements, according to the parameters, shows that supply chain decision-makers are able to estimate the optimal parameter values. While simulations allow practitioners to obtain general ideas and approximate settings, varying the lead-time is not truly possible. However, they can easily change the moving average while conducting forecasting as long as this value does not exceed the historical time horizon.

It is first important for decision-makers to further reduce inventory levels and gather additional savings. Indeed, the resulting reduction in the manufacturer translates into cost savings over time. These savings are the most important engine leading to DDI adoption. Our work shows through simulations that savings from the WMA/Newton approach exceed the savings from the SMA method. We estimate that Newton's method itself is not expensive in terms of the time implementation. While it is natural to expect a distribution of these savings between the manufacturer and the retailer, coordination is essential to achieve such improvements.

If the DDI strategy is adopted in a supply chain where actors decide to adopt WMA/Newton method, the decision-makers are faced with compromising the two major criteria axes: the forecasted mean squared errors and average inventory levels on one side, and the bullwhip effect amplifications on another side. In this paper, we have considered the enhancement of the forecast MSE and inventory level metrics since they are directly related to average inventory costs over time. The bullwhip effect is then costly to the supply chain if the upstream actor decides to base his forecasting only on the received orders process. However, in the case of the DDI strategy, the upstream actor bases his forecasting on orders and inferred demand at the same time. The knowledge of the estimated parameters and error variance interfere in the reduction of the MSE and the average inventory level at the upstream actor. If the supply chain is initially adopting a NIS strategy where the MMSE method is used in the downstream forecasts, the downstream actor is emphasized to consider a high value of N (beyond a certain break-point) in order to reduce the bullwhip effect. Else, if the supply chain is initially adopting a DDI strategy where the SMA method is used in the downstream forecasts, the subject of bullwhip effect amplification can be critical if the upstream actor decide to not use a safety stock as a buffer against orders variations. Consequently, the upstream actor needs to use a reserve inventory in order to cover the orders variations.

Except for the optimal weighting information that must be shared between the supply chain actors, this approach does not require further assumptions than those that are required by DDI with the SMA method, namely, the knowledge of the demand process (time-series structure) and its estimated parameters all along the supply chain. Thus, it is also essential to consider the costs of such coordination. These costs are primarily related to the implementation of the method itself (if another forecast method was adopted) and the weighting information sharing.

The WMA/Newton forecasting approach for the DDI strategy can be established through different managerial contracts between the manufacturer and the retailer. The literature on such contracts is abundant. For example, the manufacturer may propose contracts to the retailer based on principal agent relationships (Müller & Turner, 2005), the supply chain actors can negotiate through proposals (Dudek & Stadtler, 2005; Taghipour & Frayret, 2013). Buyback (Chen & Bell, 2011) or price discount (Jain, Seshadri & Sohoni, 2011) contracts may also be proposed.

6. Generalization for multi-level supply chains

The DDI results where the downstream actor adopts the WMA/Newton forecast approach can be extended to multi-level supply chains where there is more than two actors. Let consider a n-level supply chain where each downstream actor places an order to his formal upstream actor after revising his inventory level. We suppose that all actors accept to adopt DDI strategy through the WMA/Newton method. It means that each actor $i = 2,\ 3, \ldots, n$ will use the WMA/Newton forecast method by considering the weighting vector of his formal upstream actor $i-1$. Then every upstream actor $i = 1,\ 2, \ldots, n-1$ is able to infer the demand occurring at his formal downstream actor $i+1$. Notice that the first upstream actor is not concerned about a specific forecast approach. Fig. 5 shows a demonstration of such a multi-level supply chain.

In a set configuration such as Fig. 7, Actor 1 is generally a supplier of raw materials who endures large inventory costs. Let suppose a customer of a single product whose demand follows an $ARMA(p, q)$ process at the actor n. After revising his inventory level, this actor will place an order at the actor $n-1$. The order process will keep the same autoregressive moving average structure as the demand but it will increase its error variability, as one moves further up the supply chain. Moreover, for illustration, let consider an example of an initial customer's demand model of an $ARMA(2, 1)$ defined by:

$$D_t = 10 + 0,2\ D_{t-1} + 0,15\ D_{t-2} + \xi_t + 0,1\ \xi_{t-1}$$

where $\xi_t \to N(0,\ 1)$ is the standard normal distributed error at period t. The order process arriving at the actor $n-1$ is also an $ARMA(p, q)$ process defined by:

$$Y_t = 10 + 0,2\ Y_{t-1} + 0,15\ Y_{t-2} + \tilde{\xi}_t + 0,1\ \tilde{\xi}_{t-1}$$

where $\xi_t \sum$ is the normal error distributed error at period t, and where x_i are the Newton's weights shared by the actor $n-1$ and used by actor n in his forecasts.

We present in Table 4 the different metrics values at actor $n-1$ where NIS, DDI with SMA method and DDI with WMA/Newton method are evaluated.

Reductions of MSE and average inventory level at Table 4 is improving when moving from SMA method to WMA/Newton method. In this example, DDI with WMA/Newton allows the actor $n-1$ to reduce his average inventory level by about 39% compared to NIS and nearly 0,8% compared to DDI with SMA method. Such reductions are translatable into real inventory savings if both actors were favorable to collaborate through a benefit sharing contract. That is, we suppose that the downstream actor is favorable to such contract if he will gain a part of the savings at the upstream actor.

Table 4
NIS and DDI results for $ARMA(2, 1)$ demand process.

Adopted strategy Metrics	NIS	DDI with SMA forecasting	DDI with WMA/Newton forecasting	% of Reduction when adopting DDI with WMA/Newton rather than NIS	% of Reduction when adopting DDI with WMA/Newton rather than DDI with SMA
MSE	49.8496	19.6140	18.5583	62,7714	5,3823
$\tilde{I}_t$	17.7593	10.8756	10.7887	**39,2504**	**0,7990**

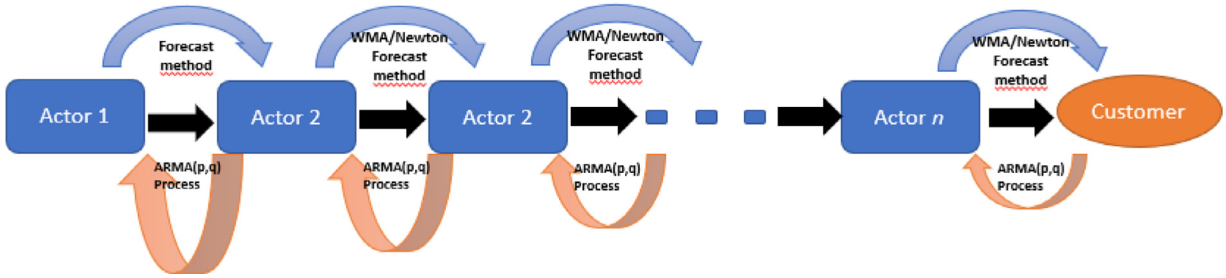

Fig. 7. N-level supply chain where actors accept DDI while adopting WMA/Newton forecasting.

In the same way, let suppose every upstream actor $i = 1, \dots, n-1$ of a supply chain propose a revenue-sharing contract to his formal downstream actor in order to convince him to adopt WMA/Newton method. Let R_i be the inventory savings of DDI adoption, at actor i. We also assume that the costs of adopting WMA/Newton are related to the Newton's weighting vector information sharing in addition of the implementation costs. Let C_i^{i+1} be the sum of the Newton's weighting information sharing cost at actor i and the implementation cost at actor $i+1$. The net profit of such collaboration is then expressed by $\pi_i^{i+1} = R_i - C_i^{i+1}$ which will be shared with the downstream actor $i+1$ according to their contract. Let β_i be the proportion of the net profit π_i^{i+1} shared with the downstream actor $i+1$, where β_i verifies $0 \le \beta_i \le 1$. Then π_i the inventory savings of the actor i, resulting from concluding two contracts of collaboration with actor $i-1$ and actor $i+1$ is expressed as follows:

$$\pi_i = \beta_{i-1}\pi_{i-1}^{i} + (1-\beta_i)\pi_i^{i+1}$$
$$\pi_i = \beta_{i-1}\left(R_{i-1} - C_{i-1}^{i}\right) + (1-\beta_i)\left(R_i - C_i^{i+1}\right)$$

Note that for actor 1, there is no actor 0 and only one contract can be established with actor 2 which implies $\beta_0 = 0$ and $\pi_0^1 = 0$. Consequently, $\pi_1 = (1-\beta_1)(R_1 - C_1^2)$. In the same manner, note that for actor n, there is no actor $n+1$ and only one contract can be established with actor $n-1$ which implies $\beta_n = 0$ and $\pi_n^{n+1} = 0$. Consequently, $\pi_n = \beta_{n-1}(R_{n-1} - C_{n-1}^n)$.

Now if we consider the whole n-level supply chain, the total supply chain inventory savings from adopting DDI strategy with the WMA/Newton method, where a revenue sharing contract is established between every successive couple of actors, is expressed as follows:

$$\pi = \sum_{i=1}^{n} \pi_i = \pi_1 + \sum_{i=2}^{n-1} \pi_i + \pi_n$$

$$\pi = (1-\beta_1)\left(R_1 - C_1^2\right) + \sum_{i=2}^{n-1}\left[\beta_{i-1}\left(R_{i-1} - C_{i-1}^{i}\right) + (1-\beta_i)\left(R_i - C_i^{i+1}\right)\right] + \beta_{n-1}\left(R_{n-1} - C_{n-1}^{n}\right)$$

The last total profit equation proves that the DDI approach with WMA/Newton forecasting improves the performance of the entire decentralized supply chain, as well as DDI with SMA method does. The enhancement is much more considerable compared to NIS strategy and it is more important than DDI with SMA method. Indeed, all the DDI with WMA/Newton forecasts outperforms the DDI results with SMA in terms of average inventory level and consequently in inventory savings. Based on our simulations, we conclude this section by the following statement:

$$\pi^{NIS} <<< \pi_{SMA}^{DDI} < \pi_{WMA/Newton}^{DDI}$$

7. Conclusion

Improving the results of supply chains coordination is one of the most important areas for academic researchers and management practitioners. Optimization presents a mathematical branch and an effective tool for collecting better management solutions. In a decentralized supply chain, actors aim to reduce their total costs by applying effective coordination approaches. One of the most cost-effective coordination approaches, namely, DDI, can be set up when actors agree to negotiate and cooperate. DDI allows the upstream actor to infer the demand of his formal downstream actors without the need for information sharing mechanisms. DDI has proved its effectiveness by obtaining almost near-optimal solutions. The literature has shown that the DDI approach cannot be applied through MMSE or SES methods for the downstream actors but only through the SMA method due to the uniqueness of the processes' propagation. Consequently, we found that it is natural to study the feasibility of DDI using other forecasting methods.

This paper is a follow-up study to previous works with the purpose of improving existing DDI results through the theoretical analysis of inventory models based on some strong assumptions. In a context of the DDI coordination strategy, instead of using the SMA method, we proposed the adoption of the WMA method combined with the well-known Newton optimization method. This paper thus enriches the existing literature by exploring the feasibility of the DDI approach when the WMA forecasting method is adopted.

We first established the expressions of the manufacturer's forecasting MSE^{DDI} and $\bar{I}^{DDI}$ and the resulting bullwhip effect. We proposed two measures, namely $BEE_{MMSE}^{WMA/Newton}$, to assess the amplification of the bullwhip effect separating the adoption of the DDI with the WMA method from the adoption of the NIS strategy with the MMSE method, and $BEE_{SMA}^{WMA/Newton}$, to assess the amplification of the bullwhip effect separating the adoption of the DDI strategy with the WMA/Newton method from the adoption of the DDI with the SMA method. Second, we mathematically formalized the MFOP and proposed the application of Newton's method for the resolution. Finally, the results for the MSE^{DDI} and $\bar{I}^{DDI}$ optimization based on the simulated causal invertible $ARMA(p,q)$ demand processes confirm the effectiveness of the WMA/Newton approach to propose further enhanced supply chain solutions.

The implications of this paper are as follows. Supply chain managers can introduce the WMA forecast method in the context of the DDI strategy because of the uniqueness of the generated orders process for upstream actors. First, the paper provides WMA/Newton as a novel approach for coordination in decentralized supply chains. This approach does not require further assumptions than those required by the DDI strategy with the SMA method, except for the optimal weighting vector, which must be shared between the supply chain actors. Second, based on the conducted simulations, the paper confirms that the DDI strategy with the WMA/Newton approach generally outperforms the NIS strategy and the DDI strategy with the SMA method in terms of MSE^{DDI} and $\tilde{I}^{DDI}$. Therefore, the paper concludes that the DDI's performance depends on the allocation vector, and especially MSE^{DDI} and $\tilde{I}^{DDI}$ generally improve with the optimal Newton's allocation. The "*generally*" statement is employed here since this work does not provide an exhaustive sensitivity analysis of the performance according to the demand process parameters. Indeed, it is not easy to check the entire sensitivity of the DDI strategy according to the combination of two sets of parameters $\phi_{i\{i=1,\ldots,p\}}$ and $\theta_{j\{j=1,\ldots,q\}}$, especially since they are not fixed in advance. Indeed, in this case, the threshold can take the form of a summation, a product, or any other linear or non-linear relationship, from which we can state a general expression of a yield threshold. Establishing such relation requires a deep study on the sensitivity according to the process parameters. Since the demand models are mathematically discrete and no continuous, there is no way to get through the partial derivative functions. We think it can be a case-by-case study to bring an exhaustive benchmark and then be able to generalize some threshold models.

Reversely, the bullwhip effect is affected. In comparison with the NIS strategy, DDI with WMA/Newton method is valuable if the parameter N is high enough vis-à-vis the lead-time L. As shown in the simulation section, a break-point from which DDI with WMA/Newton is more valuable than the NIS strategy can be determined by varying the parameter N of the forecast method. In the case of a DDI adoption where the downstream actor is favourable to switch from an initial situation of an SMA method to the WMA/Newton method, the bullwhip effect amplifies. This was predictable because of the non-equitable weights that are associated with the N historical demand observations in the method. However, the fact that the ponderation vector in the downstream actor's forecasts, is determined according to the minimization of the average inventory level of the upstream actor, results into the reduction of the mean inventory costs of the upstream actor over the time. In this case, the bullwhip effect can be costly to the upstream actor of the supply chain if he doesn't use a safety stock as a buffer against orders variations. Indeed, excess inventory can result in waste, while insufficient inventory can lead to poor customer experience and lost business. Thus, the upstream actor is emphasized to use a reserve inventory in such context of methods' change. Third, the paper concludes that supply chain managers, when the DDI with the WMA/Newton is adopted, can potentially determine the optimal parameters (N, L) in terms of MSE and average inventory levels improvements. The value of the WMA parameter N can be easily manipulated through some simulations, while managers do not truly have a large margin to vary the lead-time L. Indeed, the lead-time is often related to supply chain transportation.

From the point of a supplier or a manufacturer, the additional merit of going the extra steps of WMA/Newton method is the evident forecast MSE and inventory levels reduction which will be earned over time. The reduction of error is important because it is directly correlated to the reduction of inventory levels. As shown in equation 5, the average inventory level is a positive non-linear function of the forecast MSE. The simulated experiments in Table 3 show that all empirical inventory means resulted from adopting WMA/Newton are lower than empirical inventory means resulted from adopting SMA method. This difference may not seem significant. However, the gap percentage separating the two compared methods depends on the size of the enterprise and therefore varies from a small, medium or multinational enterprise. In addition, batch sizing rules and product structure affect the costs of a company's inventory system. (Lea & Fredendall, 2002). As example, let suppose a two-level supply chain adopting the NIS strategy where the downstream actor, a retailer, adopts the MMSE method. Moreover, let suppose that the retailer faces an ARMA(2,1) demand pattern and the average inventory level at the upstream level, a manufacturer, is equal to 1000 units. By adopting the DDI strategy where the retailer uses the SMA method, the manufacturer earns the reduction of nearly 40% of his average inventory level, let's say 400 units, and then the average inventory level is equal to 600 instead of 1000 units. In the same way, by adopting the DDI strategy where the retailer uses the WMA/Newton method, the manufacturer earns an additional average saving of 8 units plus 400 units. Hence, our work provide supplementary inventory reductions based only on the forecasting method. The Newton method's implementation is not an exhausting task. The time and resources needed for such a method depends on the capacity of qualified human resources to implementation. Moreover, the initial implementation cost is unique. We then estimate that the costs associated with the sharing of the Newton weighting vector are negligible, especially when we know that the unit holding costs of some industry products are relatively high. Indeed, if we suppose that a manufacturer produces furniture that is stored in a warehouse and then shipped to retailers, the manufacturer must either lease or purchase warehouse space and pay for utilities, insurance, and security for the location. the company is responsible for paying the salaries of the personnel responsible for moving the goods in and out of the warehouse. In addition, the company is exposed to a certain risk of damage of the goods when moving to trucks or trains for shipping. All these factors are taken into account in the calculation of the unit inventory cost. Therefore, minimizing inventory costs is an important supply-chain management strategy. The inventory presents an asset account that requires significant cash outlays. The importance of this account is then linked to the decisions made by the managers, who must minimize it in order to maintain a reasonable level of liquidity for other purposes. For example, increasing the inventory balance by 20,000 dollars means that less cash is available to operate the business each month. This situation is considered an opportunity cost. If a company wants to have more cash, it must sell its products as quickly as possible to reap its profits and move its business forward. The faster the money is raised, the more the company is able to develop its business in the short term. A commonly used indicator is the inventory turnover rate, which is calculated as the cost of goods sold divided by the average inventory (Lee, Zhou & Hsu, 2015). For example, a company that has 1 million dollars in cost of goods sold and an inventory balance of 250,000 has a turnover ratio of 4. The goal is to increase sales and reduce the required amount of inventory so that the turnover ratio increases. By projecting our results of simulations on this indicator, the turnover ratio of the manufacturer where the retailer uses the WMA/Newton method, is higher than that where the retailer uses the SMA method, because the average inventory level in the first case is lower than that in the second case $\tilde{I}^{DDI}_{WMA/Newton} \leq \tilde{I}^{DDI}_{SMA}$ for the same fixed cost of goods sold. Consequently, this capability of reducing the average inventory level and increasing the turnover ratio is one of the most important catalysts of an enterprise to enhance productivity and competition. As it was argued in this paper, some typical contracts can be proposed by the upstream actor to his formal downstream actor, in order to collaborate with the aim of creating common and shared opportunities of trust, transparency and future coordination.

The SMA method is preferable against the WMA method in terms of bullwhip effect. However, The SMA method is not preferable in terms of the distribution of the inferred demand when compared against the WMA method, because the results of the SMA method present one specific case of the results of the WMA method. Indeed, if we replace x_i by $1/N$ for $i = 1, \ldots, N$ in all the expressions where WMA is used, we exactly retrieve all the expressions where SMA is used for forecasts. It's then concluded that there is no preference between SMA and WMA in terms of the distribution of the inferred demand.

We conclude our paper with natural lines for future studies. First, the DDI strategy can still be evaluated using other forecasting methods. Second, it would be interesting to adopt the minimization of the bullwhip effect as the objective function of the WMA/Newton approach. Another direction is the consideration of multiobjective optimization for parallel improvements of the supply chain performance metrics.

Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.ejor.2020.04.044.

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